Transport Characteristics of Suspensions: II. Minimum Transport Velocity for Flocculated Suspensions in Horizontal Pipes

DAVID G. THOMAS

Oak Ridge National Laboratory, Oak Ridge, Tennessee

The minimum transport velocity (defined as the *mean stream velocity* required to prevent the accumulation of a layer of stationary or sliding particles on the bottom of a horizontal conduit) has been determined for flocculated thorium oxide and kaolin suspensions flowing in glass pipes. The pipes ranged from 1 to 4 in. in diameter, and the concentration was varied from 0.01 to 0.17 volume fraction solids.

Two flow regimes were observed depending on the concentration of the suspension. In the first the suspension was sufficiently concentrated to be in the compaction zone and hence had an extremely low settling rate. The second regime was observed with more dilute suspensions which were in the hindered-settling zone and settled ten to one-hundred times faster than slurries which were in compaction. The concentration for transition from one regime to the other was dependent on both the tube diameter and the degree of flocculation. The suspension particles were smaller than the thickness of the laminar sublayer, and they settled according to Stokes' law for the particular conditions of this study. Under these circumstances the relation developed for dilute suspensions is consistent with particle transfer in the radial direction owing to Bernoulli forces on the particle and the action of turbulent fluctuations which penetrate the laminar sublayer. For concentrated suspension in compaction the minimum transport velocity was given by a characteristic critical Reynolds number.

Particulate matter may be conveyed through horizontal pipes with no loose deposits on the bottom of the pipe, provided the mean stream velocity exceeds a certain value designated as the *minimum transport velocity*. If this velocity is not exceeded, solids deposition will occur, resulting in a decrease in circulating stream concentration and ultimately in a flow blockage. Since it is absolutely essential that such occurrences be prevented in suspension transport systems (such as large-scale slurry pipelines and in many of the proposed fluid fuel reactor concepts), an investigation was undertaken of the factors affecting the minimum transport velocity.

Flowing suspensions may be divided into two categories depending on whether the solids are homogeneously distributed or whether there is a heterogeneous mixture with the concentration at the top of the pipe being less than that at the bottom. Extensive theoretical and experimental studies (*1*) have shown that the major factor affecting distribution of suspended solids in a flowing stream is the ratio of the terminal settling velocity to the wall shear velocity (U_t/u_{*w}). All expressions have the form

$$\left(\frac{c_2}{c_1}\right) = \Psi\left[\left(\frac{Z_2}{Z_1}\right)^{-U_t/\kappa u_{*w}}\right] \quad (1)$$

with c_1 and c_2 being the concentration at elevations Z_1 and Z_2, respectively. The coefficient κ is commonly (*2*) considered a universal constant for the flow of homogeneous, Newtonian fluids with a value of 0.4. However sediment transport data (*3*) indicate that it may have a value as low as 0.19 in two-phase flow. The wall shear velocity is given by the expression $u_{*w} = \sqrt{g_c \tau_w/\rho_m} = \sqrt{g_c D\Delta p/4 \rho_m L}$ and hence is a function of both the flow rate and of the system characteristics. The terminal settling velocity is the settling rate of a single particle in the fluid medium and is a function of the particle diameter and density as well as the viscosity and density of the fluid medium.

The present investigation was confined to systems in which the particle size (hence the settling rate) was sufficiently small to insure homogeneous flow of the suspension, in accordance with Equation (1). Particles in this size range settle according to Stokes' law, and since the smallest particles approach colloidal dimensions, the suspensions may be flocculated with non-Newtonian flow characteristics. Previous studies (*4*) of heat and momentum transfer characteristics of non-Newtonian aqueous thorium oxide suspensions have shown that the Bingham plastic laminar flow model provided a satisfactory basis for the correlation of data. The laminar flow constants determined by application of this model will also be used in the present study of flocculated suspensions.

Although little data have been published giving the minimum velocity required to transport flocculated suspensions, Durand (*5*) has stated, without providing supporting evidence, that the minimum transport velocity is given by

$$\frac{DV_{MT}\rho}{\mu_e} = \frac{D\,V_{MT}\,\rho}{\eta\left[1 + \dfrac{g_c\,D\,\tau_y}{6\,\eta\,V_{MT}}\right]} > 2{,}000 \quad (2)$$

with the effective viscosity determined by the use of the Bingham plastic model. Subsequent studies (*6, 7*) with flocculated suspensions have shown that under some conditions (*6*) Reynolds numbers as high as 2.9 to 3.6 x 10^4 are required to prevent solids deposition.

The object of this investigation was to determine the range of validity of Equation (2), obtain a correlation for use when Equation (2) is not applicable, and establish a procedure for differentiating between the two flow regimes. The approach selected was to relate direct observation of the minimum transport velocity to the more easily measured hindered-settling rate and the laminar and turbulent flow characteristics of the suspension.

BASIC CONCEPTS

Before proceeding with the description of the flow phenomena which occur at the minimum transport condition it is necessary to describe some characteristics of small-particle systems. Suspensions of particles having near-colloidal dimensions are often observed to be flocculated. In a flocculated suspension the particles stick together in the form of loose, irregular clumps in which the original particles can still be recognized (*8*). The clusters may be disrupted under shear-flow conditions but reform quickly in the absence of shear. However under hindered-settling conditions the flocs appear to settle as more or less discrete clumps. Thus comparison of the results of hindered-settling measurements of flocculated suspensions with theoretical and empirical studies of hindered settling of noninteracting particles gives a measure (*9*) of the floc diameter and density under low-shear conditions. Suspensions with concentrations sufficiently large for the clusters to be in contact are observed to have a very low settling rate. Such a system is said to be in compaction, since fluid must be squeezed from the clusters in order for settling to occur. Highly flocculated suspensions may be in compaction at volume fraction solids as low as 0.05 in contrast to values (*10*) greater than 0.60 for random packing of noninteracting particles. The extremely porous floc structure arises (*8*) because, once the particles touch, they adhere to each other in a haphazard manner owing to forces of a colloidal nature. Because of their very low settling rates, highly flocculated suspensions which are in compaction may often be regarded as homogeneous fluids with a unique set of physical properties.

Flowing suspensions are considered to exceed the minimum transport velocity only under those conditions which prevent the accumulation of a layer of stationary or sliding particles on the bottom of the conduit. This requires momentum transfer between the main flowing stream and the fluid elements adjacent to the conduit wall so that a radial movement of these fluid elements is produced. It has been suggested (*11, 12*) that the radial momentum transfer may occur through either velocity or pressure fluctuations in the radial direction, but in either event fluctuating turbulent flow is required. This represents the first condition necessary for the establishment of the minimum transport velocity.

Thus the minimum transport velocity of a flocculated suspension in compaction (in which the suspension may be considered as a homogeneous fluid with loose clumps of particles in contact throughout the liquid) corresponds just to the initiation of a radial component of momentum transport or simply to the transition from laminar to turbulent flow. This is the criterion proposed by Durand (*5*), Equation (2). However it is not clear that the value of the critical Reynolds number should be as low as 2,000 as he proposed because it is desired to specify the condition for turbulent flow, not for departure from laminar flow. Note that there is no a priori reason for these two values to coincide; indeed the transition region has been observed to extend to quite large Reynolds numbers. Consequently the value of the numerical term in Equation (2) may also be much larger than the recommended value of 2,000.

For more dilute suspensions in the hindered-settling range the flocs are no longer in contact. Hence energy must be supplied in excess of that required to initiate turbulence in order to overcome the tendency of the flocs to settle. Thus the criterion proposed above would no longer be sufficient; instead the minimum transport condition is expected to be a function of both the floc settling velocity and the intensity of turbulent fluctuations, as well as other particle and fluid properties. The settling rate provides a measure of the tendency of the particles to settle out, whereas the turbulent fluctuations provide a driving force to maintain the particles in suspension.

The measurement of the turbulent fluctuations of suspensions is beyond the scope of the present investigation. However Laufer (*13*) has shown that the turbulent fluctuations of Newtonian fluids at any given radial position is proportional to the wall shear velocity. Further, his results showed that the ratio of the root mean-square fluctuating velocity to the wall shear velocity was essentially independent of the mean stream Reynolds number, provided the distance from the wall was expressed in terms of the dimensionless parameter $y^+ = yu_{*w}\,\rho/\mu$. Since the fluctuations must be zero at the wall, the data in the vicinity of the wall may be fitted with a power series in y^+. In order to make use of these facts it is necessary to consider the limiting case of minimum transport at infinite dilution. Although this introduces the additional complication of determining the effect of the solids concentration on the wall shear velocity required for particle transport, it leads to a more useful result in the long run since the wall shear velocity is easily related to the pressure drop and flow rate, parameters that are important in system design.

When one recalls that the ratio of particle terminal settling velocity to wall shear velocity is the principal factor affecting the distribution of solids in a flowing fluid [Equation (1)], and proceeds on the basis that turbulent fluctuations in the vicinity of the wall may be fitted with a power series in y^+, then dimensional considerations suggest that the minimum transport condition may be given by an expression of the form

$$\frac{U_t}{(u_{*w})_{MT}} = K\left[\frac{D_p\,(u_{*w})_{MT}\rho}{\mu}\right]^b \tag{3}$$

where the coefficient and exponent are to be determined experimentally and the length term is taken as the particle diameter in order to give a suitable scale.

The critical concentration at which Equation (3) is no longer controlling but at which suspension compaction provides the controlling mechanism is expected to be a function of both the diameter of the conduit and the degree of flocculation. The conduit dimensions enter because the flocs are supported in part by the walls and bottom of the container.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

Minimum transport velocities were determined in a loop system having three 40 ft. long sections of 1.045–, 2.056–, and 3.97-in.–diam. pipe. The last 10 ft. of each section consisted of glass pipe where the minimum transport condition was determined visually. Mean stream velocities were measured with a weigh tank, and the suspension was circulated with a pump with a variable speed drive. A by-pass line around the test section gave additional flexibility in adjusting the velocity.

All of the solids in the system were suspended completely before the start of any minimum transport velocity determination. The mean velocity was then lowered until the first sediment was observed sliding along the bottom of the conduit. The velocity was then increased to resuspend the sediment and lowered again to a value slightly higher than when the sliding bed was observed. This procedure was followed until the system had operated for at least 30 min. with no sliding bed and with the suspension being transported by saltation. The mean stream velocity for this condition was defined as the minimum transport velocity and was reproducible to within $\pm 5\%$.

Previous studies (*4*) have shown that a capillary tube viscometer with a ⅛-in.–I.D. tube and an L/D ratio of 1,000 gives laminar flow physical-property data for non-Newtonian suspensions that are suitable for correlating turbulent flow momentum transfer data. All of the laminar flow data taken with the flocculated suspensions used in this study were measured with such a viscometer, and the data were fitted satisfactorily with the Bingham plastic model. In addition, turbulent flow pressure-drop measurements were made with each material over a wide range of flow rates and concentrations in both ⅜- and 1-in.–I.D. tubes to enable calculation of wall shear velocities directly from experimental data. The results of the measurements fitted the correlations presented earlier (*4*) in connection with a heat and momentum transfer study and support the belief that these correlations are sufficiently general for application to any flocculated suspension system.

Minimum transport velocities were determined for concentrations from 0.01 to 0.17 volume fraction solids, a range sufficient to include both the hindered-settling and the compaction regions. Two different thorium oxide preparations and a single kaolin powder were used in these tests. Particle sizes were determined with a modified Andreisen method (*14*) and 0.005 M sodium pyrophosphate as a dispersant. The size distributions all followed a logarithmic normal distribution with the parameters given in Table 1. In order to further characterize the solids used in the present study this table also includes particle surface areas determined by nitrogen adsorption at liquid nitrogen temperatures as well as true particle densities. Finally the table gives particle characteristic data from all other studies of suspension transport which reported sufficient data to permit application of the correlations developed in this investigation.

EXPERIMENTAL RESULTS

Flowing suspensions may be classified according to the manner in which the solids are distributed in the flow-

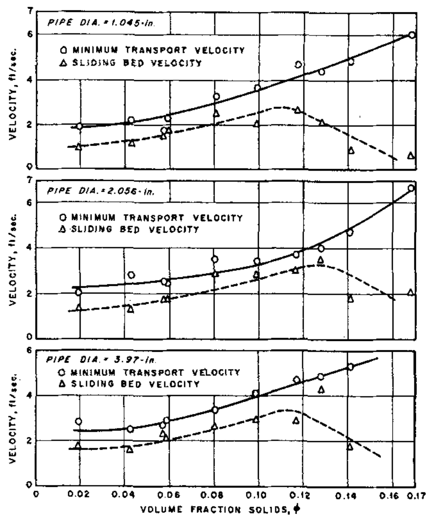

Fig. 2. Effect of suspension concentration on minimum transport and initial sliding-bed velocity of kaolin-water suspension.

ing stream. Three flow regimes were observed in the tests with flocculated suspension:

1. High-velocity flow in which the solids were transported while homogeneously distributed throughout the flowing fluid. If the ratio of terminal settling velocity to wall shear velocity becomes sufficiently large [see Equation (1)], then a heterogeneous distribution may be observed even though all of the material is completely in suspension. However the settling rate of the flocculated suspension in these tests was always so small that as the suspension velocity was decreased, the minimum transport velocity was reached before the wall shear velocity was reduced sufficiently to permit a heterogeneous flow distribution. In order to verify the homogeneity of concentration the mean stream velocity was set at a value 10% greater than the minimum transport velocity. Then four samples were withdrawn at two different sampling rates from each of eight uniformly-spaced radial holes of 1/16-in diameter; at the same time mean stream samples were collected from the discharge of the pipe. A statistical analysis of the data, with the standard t test for the difference of means, showed that there was a 99% probability that there was no difference in the mean concentration as determined from each of the radial sampling positions and the mean determined by collecting samples from the outlet of the pipe. This was true for both sampling rates. Therefore within the precision of the analytical data ($\pm$ 3%) flow was considered to be homogeneous for velocities greater than the minimum transport velocity.

2. Flow at velocities less than the minimum transport velocity in which all the material was being transported, but a continuous filament of sliding bed was observed. The highest velocity for which this occurred was termed the "sliding bed velocity." Further reduction in velocity permitted solids deposition and, under certain circumstances, flow blockage. This is illustrated in Figure 1 for a particular suspension having a settling rate of 3.3 x 10^{-3} ft./sec. In this test the bypass line was partially open so that as solids deposition occurred in the test section, the flow rate through the bypass could increase. The flow-rate measurements were made by repeated rapid operation of the weigh tank, and only the flow through the test section was measured. The minimum transport velocity was 2.76 ft./sec.; as the mass flow rate was slowly reduced, a continuous sliding bed was first observed at 2.08 ft./sec. Further reduction of the velocity to 1.60 ft./sec. merely increased the thickness of the sliding bed with no evidence of a flow blockage in 30 min. of operation. However reduction of the velocity from fully suspended flow to 1.54 ft./sec. gave evidence of flow blockage within 10 min., and flow was completely blocked within 30 min. Further, a reduction of the velocity from fully suspended flow to 1.37 ft./sec. produced a flow blockage in just 10 min. Thus flow block-

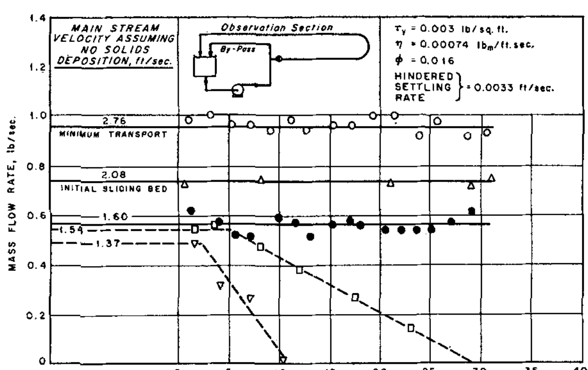

Fig. 1. Effect of flow rate on nature of suspension transport showing flow blockage due to solids deposition at low flow rates.

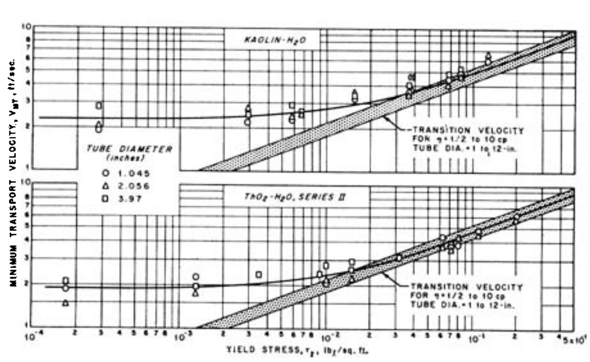

Fig. 3. Effect of yield stress on minimum transport velocity showing that velocity for transition from laminar to turbulent flow equals the minimum transport velocity at large values of the yield stress.

ages are possible in relatively short time, even though the mean stream velocity was over 400 times the settling rate. It is also of interest to note that the conditions under which the first flow blockage was observed correspond exactly to the situation that the suspension could settle a distance of one pipe diameter during the length of time required to flow from the entrance to the exit of the longest straight run of pipe. However insufficient data were taken to generalize this observation.

A typical example of the effect of solids concentration on the minimum transport and the sliding bed velocity is shown for the flocculated kaolin suspension in Figure 2. Similar curves have been observed (*15*) qualitatively for nonflocculated suspensions having particles with mean diameters from 100 to 6,000 μ. In both cases the minimum transport velocity increased continuously with concentration, and the decrease in sliding bed velocity at large concentrations was associated with flow blockages.

The minimum transport velocity is shown in Figure 3 as a function of the experimentally determined values of the yield stress for flocculated suspensions of kaolin and thorium oxide. Tube diameter had no significant effect on the minimum transport velocity and the yield stress had little effect for values less than 0.003 lb.-force/sq. ft. For values greater than this the minimum transport velocity approached and finally coincided with the velocity for the transition from laminar to turbulent flow represented by the cross-hatched area on the figure. Previous studies (*9*) have pointed out that for many non-Newtonian suspensions the yield stress is the primary factor affecting the transition velocity, with the tube diameter and coefficient of rigidity having only a secondary effect. Hence it is not surprising that the minimum transport velocity was almost a unique function of the yield stress.

TABLE 2. EFFECTIVE REYNOLDS NUMBER FOR SUSPENSION TRANSPORT

	Diameter, in.	Effective Reynolds number, Experimental suspension transport	Calculated for transition, Lower limit of concentration	Calculated for transition, Upper limit of concentration	Ratio, exp./avg. calc.
ThO_2-I	1.045	3,500	2,140	2,500	1.51
	2.056	3,000	2,650	2,760	1.11
ThO_2-II	1.045	3,100	2,160	2,820	1.25
	2.056	3,200	2,450	3,070	1.16
	3.97	3,500	3,080	3,330	1.09
Kaolin	1.045	3,500	2,030	2,390	1.58
	2.056	3,600	2,440	2,730	1.39
	3.97	4,800	2,790	3,100	1.63

The data shown in Figure 3 confirm that for sufficiently high concentrations the minimum transport velocity and the velocity for transition from laminar to turbulent flow are substantially the same. However this result remains to be generalized, and the range of concentration for which it is valid must be demonstrated.

Suspensions in Compaction

For suspensions in compaction a simple analysis showed that the minimum transport condition should occur upon establishment of a radial component of motion of the fluid elements adjacent to the wall. This should occur as the effective Reynolds number $N_{Re} = DV\rho/\eta(1 + g_c\, D\, \tau_y/6\, \eta V)$ exceeds a critical value, the magnitude of which was expected to be from 2,000 to 6,000. Typical data are shown in Figure 4 as a function of volume fraction solids. Notable features are: effective Reynolds number for minimum transport as large as 60,000 were observed with dilute suspension, as the concentration was increased the effective Reynolds number decreased to a constant value in the compaction zone, the critical value for $(N_{Re})_{MT}$ in the compaction zone was between 2,500 and 5,500 for the particular suspensions studied, and the concentration at which compaction occurred [that is at which $(N_{Re})_{MT}$ = constant] increased with tube diameter.

Critical Concentration for Compaction. In order to obtain additional evidence on the effect of tube diameter and concentration on the onset of compaction a series of hindered-settling studies were made for each suspension. A typical set of data are shown in Figure 5. For sufficiently dilute concentrations the hindered-settling rate was independent of tube diameter, decreasing regularly as the concentration was increased. However for any given tube diameter there was a certain critical concentration beyond which the settling rate decreased sharply to a value one-tenth to one-fiftieth that expected if there were no wall effect. This decrease in settling rate was due to the suspension being in compaction and, as shown by the family of lines on Figure 5, there was a regular increase of the critical concentration with tube diameter. The critical concentration was also a function of the degree of flocculation, and hence the diameter-concentration relation must be determined for each particular suspension.

The critical compaction concentration for each suspension and tube diameter determined from hindered-settling measurements is compared with similar values determined from minimum transport studies in Figure 6. The good agreement observed indicates that critical concentrations determined from a simple series of hindered-settling tests can be used to determine the range of applicability of minimum transport relations developed for the compaction region.

Critical Reynolds Number for Suspension Transport. The data in Figure 4 show that the effective Reynolds number for suspension transport is constant for concentrations greater than the critical value for compaction. Experimental values of the effective Reynolds number for suspension transport for concentrations larger than the critical are given in Table 2. This table also includes values of the effective Reynolds number for the transition from laminar to turbulent flow calcu-

TABLE 1. PHYSICAL PROPERTIES OF SOLIDS USED IN SUSPENSION TRANSPORT STUDIES

	Particle characteristics, Size distribution on weight basis, Mean diameter, D_p, μ	Logarithmic standard deviation, σ	Density, g./cc.	Nitrogen surface area, sq. m./g.	Settling rate in water, ft./sec.
ThO_2-I*	2.0	2.4	10.0	14.0	6.6×10^{-5}
ThO_2-II*	0.74	2.6	10.0	18.0	9.0×10^{-6}
Kaolin*	2.85	4.0	2.65	7.6	1.3×10^{-4}
Glass (*21*)	66	~1.3	2.77	NA†	1.1×10^{-2}
Tungsten (*6*)	3.5	3.4	18.9	NA	4.0×10^{-4}
Coal (*20*)	23	1.7	1.35	NA	1.2×10^{-1}**

* Present study.
† NA = not available.
** Settling rate in air.

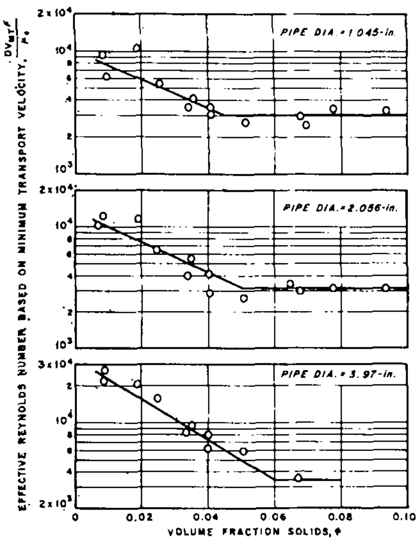

Fig. 4. Effect of concentration on minimum transport Reynolds number of thorium oxide-water suspension, series II.

lated by procedures previously recommended (*4*). Briefly, the different steps required for prediction of the onset of transition flow are:

1. Calculate the value of the Hedstrom number $N_{He} = g_c\ \rho\ \tau_y\ D^2/\eta^2$ and identify its location on a Fanning friction factor-Reynolds number $(DV\rho/\eta)$ plot containing the Hedstrom number grid as a parameter.*

2. Locate the turbulent-flow friction-factor line on the same plot by

$$f = 0.079 \left(\frac{\mu}{\eta}\right)^{0.48} (N_{Re})^{-0.25(\mu/\eta)^{0.15}} \quad (4)$$

The applicability of this expression for calculating friction factors for the present data was demonstrated by extensive measurements over the complete concentration range with the three different suspensions

3. The intersection of these two curves gives the critical Reynolds number $DV\rho/\eta$ for the onset of transition flow. This Reynolds number can be converted to the desired effective Reynolds number by substituting the effective viscosity

$$\mu_e = \eta \left[1 + \frac{g_c\,D\,\tau_y}{6\,\eta\,V}\right] \quad (5)$$

for the coefficient of rigidity.

The transition Reynolds numbers were calculated at the upper and lower limits of concentration for which each suspension was in compaction; both values are included in the table. There is very little variation of transition Reynolds numbers with concentration for any given suspension and tube diameter. However there is a systematic increase of transition Reynolds number with tube diameter, values as large as 3,000 being calculated for the 4-in. pipes.

Comparison of the calculated transition Reynolds numbers with the experimentally observed minimum transport Reynolds numbers shows that the latter are consistently 10 to 60% larger, instead of being equal as predicted (*5*). Although the reason for this is not known unequivocally, one explanation may be associated with the nature of the transition region. Flow in the transition region has been shown to consist of alternate slugs of laminar and turbulent fluid (*16*). Since there is no radial motion in the laminar slugs, the minimum transport condition would not be expected to occur at the Reynolds number for departure from laminar flow (as calculated and presented in Table 2) but at the end of the transition region where the flow is no longer intermittently turbulent. Qualitative observation with Newtonian fluids (16) and with flocculated suspensions (*17*) suggest that the transition process is a function not only of the nature of the entrance region but also of an unknown [possibly structural (*16*)] physical parameter of the fluid aside from the viscosity and density. Because of the difficulty in making an exact prediction of the end of the transition region it is recommended that the effective Reynolds number for the minimum transport condition be taken as 1.6 to 4 times the value of the transition Reynolds number as determined by the procedure given above. The larger recommended numerical value corresponds to the upper limit on the extent of the transition region determined from non-Newtonian suspension heat transfer measurements (*4*). The smaller numerical factor represents the upper limit on the transition region as determined in the present study.

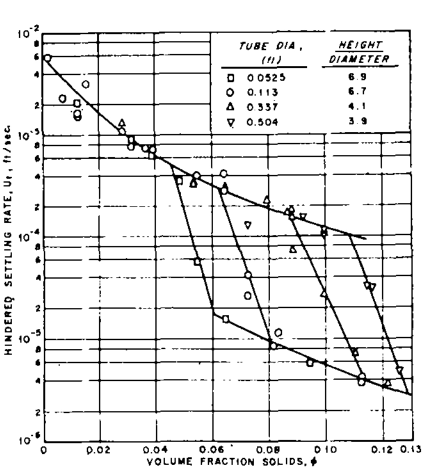

Fig. 5. Hindered settling rate as a function of suspension concentration for flocculated thorium oxide suspension (series I) showing effect of container diameter on onset of compaction.

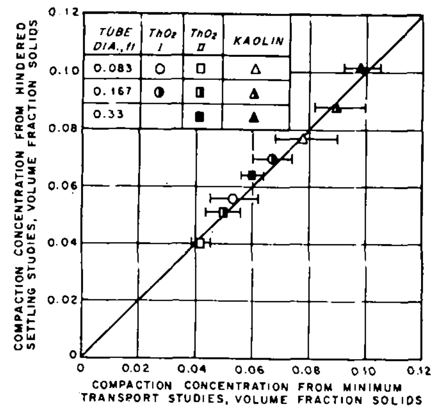

Fig. 6. Comparison of compaction concentrations determined from hindered settling and from minimum transport studies.

SUSPENSIONS IN THE DILUTE-HINDERED-SETTLING REGION

Suspensions which are sufficiently dilute no longer act as a homogeneous mixture with the flocs in contact but as a two-phase mixture with the flocs free to settle. Under these circumstances the minimum transport condition is expected to be primarily a function of the floc settling rate and the intensity of turbulence. As pointed out above it was beyond the scope of the present investigation to determine turbulent fluctuations. The alternate procedure selected was to relate the minimum transport condition to the wall shear velocity, a parameter that has been shown (*13*) to be directly proportional to the turbulent fluctuations at any given radial position.

The wall shear velocity at the minimum transport condition is shown in Figure 7 as a function of volume fraction solids for the same thorium oxide suspension data presented previously in Figure 4. The values of the wall shear velocity were calculated from direct measurement of the pressure drop with the expression $u_{*w} = \sqrt{g_c\,\tau_w/\rho_m} = \sqrt{g_c D\,\Delta p/4\,\rho_m\,L}$, or alternatively the value of the shear stress τ_w was obtained from Equation (4) with experimentally determined values of the laminar flow properties together with the minimum transport velocity determined from mass-flow-rate measurements. For mean stream velocities greater than 4 ft./sec. the maximum deviation in the value of the wall shear velocity calculated from the two procedures was less than 4%, and for all the data the mean deviation was less than 5%.

At low volume fraction solids the wall shear velocity was nearly constant; however a slight concentration

* Data for construction of a Hedstrom plot and the original data for the minimum transport velocity have been deposited with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., as document 6621, and may be obtained for $1.25 for photoprints or for 35-mm. microfilm.

TABLE 3. EFFECTIVE FLOC CHARACTERISTICS FOR MINIMUM TRANSPORT STUDIES

	Floc diameter, μ	Floc density, g./cc.	Settling rate extrapolated to zero concentration, ft./sec.
ThO_2-I	56	2.09	5.9×10^{-3}
ThO_2-II	27	1.96	9.3×10^{-4}
Kaolin	22	1.40	3.7×10^{-4}
Tungsten*	25	6.4	8.0×10^{-3}

* Floc characteristics were determined in present study with the use of powder having particle size distribution similar to that given in reference 6.

dependence was observed at volume fraction solids greater than 0.03 to 0.06. Further, at the low volume fraction solids the minimum transport wall shear velocity was independent of tube diameter. Hence the wall shear velocity at infinite dilution, required for the subsequent correlation, was obtained by a simple extrapolation of the concentration independent portion of the curves, the resulting value being essentially independent of pipe diameter in the range 1 to 4 in.

Effective Particle Diameter

Before proceeding with the minimum transport correlation as given by Equation (3) one must select a particle diameter and determine the settling rate for this particle. This choice is complicated not only because of the size range of the individual particles, as shown in Table 1, but also because of the flocculated nature of the suspension. One factor assisting the selection is the primary assumption inherent in Equation (3), that is an extrapolation to zero concentration. This means that the proper settling rate is that for the individual particle rather than the hindered-settling rate for many particles. For flocculated suspensions the range of values for the effective particle diameter extends from that for individual particles in regions of intense turbulence to that for large flocs in regions of low shear. Recalling that the minimum transport condition corresponds to the low-shear situation in which the tendency for the particle to settle is just counterbalanced as the particle approaches the wall, one can see that a useful particle diameter can be obtained from a series of settling-rate measurements with flocculated suspension such as those shown in Figure 5. These measurements, extrapolated to zero concentration, represent a close approximation to the minimum transport condition in the vicinity of the wall and in any event give an upper limit for the effective particle or the floc diameter. The procedure for determining floc diameter has been described previously (*18, 19*). Briefly the procedure is:

1. Determine the settling rate of the flocculated suspension as a function of concentration.
2. Determine the ratio of hindered-settling rate at a known concentration to the hindered-settling rate at zero concentration.
3. Determine the effective volume fraction solids to give the same reduction in settling rate as in step 2, on the assumption that the suspension was composed of solid noninteracting spherical particles. Many semiempirical and theoretical relations have been developed to describe this relation (*9, 18*).
4. A simple material balance with known particle and fluid densities and the two values of volume fraction solids determined in steps 2 and 3 give the floc density.
5. Combination of the settling rate, extrapolated to zero concentration, with the floc density defines a unique floc diameter.

Values for the materials used in the present study are given in Table 3. Comparison of the data from this table with that from Table 1 shows that there is a marked difference between the characteristics of the mean single particle and of the average floc. In addition it is apparent that floc characteristics are not simple multiples of particle characteristics, so it would not be possible to substitute one for the other in the final correlation by the simple expediency of adjusting a numerical constant.

Minimum Transport Correlation for Dilute Suspensions

When one returns to the minimum transport correlation, the present data are plotted as $(u_{*w})_{MT}/U_t$ vs. $D_p (u_{*w})_{MT} \rho/\mu$ in Figure 8. This figure also includes data from other investigations of fine-particle transport (*6, 20, 21*). The range of data included in Figure 8 is given in Table 4 together with the variability, that is the ratio of the largest to the smallest value of each variable. The wide range in variability of the data (greater than ten fold except for the particle diameter and density) precludes the possibility of one variable exerting a dominant influence on the correlation. In accord with the limitations prescribed at the outset the figure shows the range of applicability of Stokes' law and the thickness of the classical laminar sublayer with the length term D_p evaluated by taking the wall as the origin. A least-squares analysis of the data gave a value of 2.61 for the exponent and 0.0083 for the coefficient of Equation (3).

Consideration of the modes of momentum transfer in the vicinity of the wall show that Bernoulli forces accompanying the velocity gradient and turbulent fluctuation which penetrate the laminar sublayer are both significant factors in particle transport. The contribution of these forces can be evaluated in the form of Equation (3) if the analysis is restricted to limiting conditions of zero concentration and transport along the bottom surface of the conduit. The limitation on flow geometry represents the most severe condition for transport through horizontal conduits, since deposits on inclined and upper conduit surfaces would be removed by combined gravitational and hydrodynamic forces, whereas transport of material on the bottom requires the gravitational force to be overcome by hydrodynamic forces.

Lift Due to Bernoulli Forces. A sphere, placed in a flowing stream having a velocity gradient, experiences a force due to the accompanying pressure difference as given by Bernoulli's equation. A simple analysis (*22*), in which the sphere is replaced by a source and a sink of equal strength infinitely close together, shows that the resultant force is in the direction of the pressure gradient and that the magnitude of the force is 3/2 the product of the volume of the sphere and the pressure gradient. Combining this expression with that for the gravitational force on a particle one obtains

$$\frac{F_l}{F_g} = \frac{3\,\rho\,(u_1^2 - u_2^2)}{4\,g_L\,D_p(\rho_p - \rho)} \qquad (6)$$

TABLE 4. RANGE OF VARIABLES IN DILUTE SUSPENSION CORRELATION

	Smallest value	Largest value	Variability, largest/smallest
Particle diameter, μ	22	66	3.0
Pipe diameter, in.	1	12	12
Fluid density, lb.-mass/cu. ft.	0.08	62	780
Particle density, lb.-mass/cu. ft.	84	400	4.8
Fluid viscosity, centipoise	0.018	1	55
Pressure gradient, (lb.-force/sq. ft.)/ft.	0.094	2.8	30
Mixture density, lb.-mass/cu. ft.	0.11	74	670

An identical expression (23) is obtained for the lift experienced by a rotating sphere in a uniform flow field except that the numerator on the right side of the equation contains a lift coefficient C_l which has an experimentally determined value which varies from 0.2 to 0.4, depending on the rotational velocity of the sphere. For generality the lift coefficient will be retained in future expressions. Combining Stokes' law

$$U_t = \frac{g_L D_p^2(\rho_p - \rho)}{18\ \mu} \quad (7)$$

and the expression for the velocity gradient in the laminar sublayer

$$\frac{u}{u_{*w}} = u^+ = y^+ = \frac{y\, u_{*w}\, \rho}{\mu} \quad (8)$$

with Equation (6) one obtains

$$\frac{U_t}{u_{*w}} = \left(\frac{C_l F_g}{F_l}\right) \times \frac{1}{96}\left(\frac{D_p u_{*w} \rho}{\mu}\right)^3 \quad (9)$$

This expression is plotted in Figure 8 as curve *a*, on the assumptions that the ratio of $(C_l F_g/F_l) = 1$ [in accord with the results of Munk's (22) analysis for a nonrotating sphere] and that particle transport occurs when the ratio $F_l/F_g = 1$. The agreement with the experimental data is quite satisfactory, the value of the exponent being somewhat larger than the value obtained empirically.

Phenomenological Analysis. Turbulent fluctuations which penetrate the laminar sublayer provide a driving force for the diffusion of particles toward the turbulent core (*11*). The existence of such eddies has been demonstrated experimentally (*24*); however their magnitude has not been measured. An estimate of the magnitude can be obtained with Deissler's expression (*25*) for the eddy diffusivity in the vicinity of the wall, Equation (10)

$$\epsilon = n^2 u\, y \left(1 - e^{-\frac{n^2 u\, y}{\mu/\rho}}\right) \quad (10)$$

where the constant n was determined experimentally to be 0.124. Equation (10) may be written in the same form as Equation (3) by expanding the exponential term, neglecting the higher order terms in n, and making the following assumptions:

1. $\tau = \tau_w$ in the vicinity of the wall,

2. $\overline{u'\ v'} = \epsilon \dfrac{du}{dy}$,

3. $u^+ = y^+$ [Equation (8)].

This gives

$$\left[\frac{\overline{u'\ v'}}{u_{*w}^2}\right]^{1/2} = n^2 \left[\frac{y\, u_{*w}\, \rho}{\mu}\right]^2 \quad (11)$$

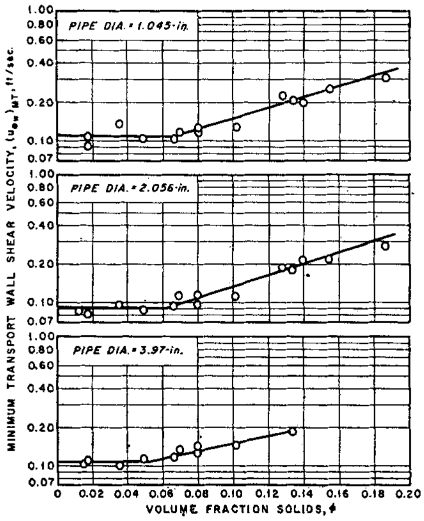

Fig. 7. Effect of concentration on minimum transport wall shear velocity for thorium oxide-water suspension.

If, in addition, the u' and v' fluctuations are equal and there is no slip between the particle and the liquid, then the minimum transport condition is observed when the turbulent fluctuations at a distance of one particle diameter from the wall [calculated from Equation (11)] are equal in magnitude to the particle settling rate. The curve based on these assumptions is labelled *b* on Figure 8; the agreement with the data is quite good, the value of the exponent being somewhat smaller than the empirical value.

DISCUSSION OF DILUTE SUSPENSION CORRELATION

The minimum transport correlation for dilute suspensions

$$\frac{U_t}{(u_{*w})_{MT}} = 0.0083 \left(\frac{D_p\ (u_{*w})_{MT}\ \rho}{\mu}\right)^{2.61} \quad (12)$$

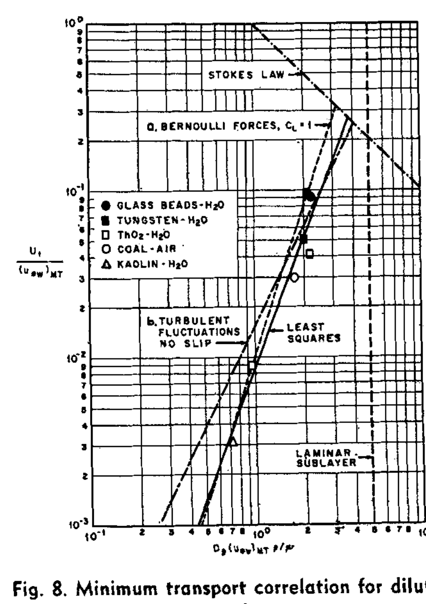

Fig. 8. Minimum transport correlation for dilute suspensions.

was developed for the limiting case of zero concentration. However, examination of Figure 7 shows that the minimum transport wall shear velocity is essentially constant as the volume fraction solids is increased from 0 to 0.03 to 0.06. The data of Figures 2 to 4 indicate that the transition from dilute to compaction type of behavior is smooth and continuous. This means that for flocculated suspensions of fine particles Equation (12) is valid virtually up to the compaction concentration. In cases of questionable validity the minimum transport condition can be determined by the compaction region relationships and the most conservative value selected.

Although Equation (3) was based on dimensional considerations, an analysis based on simple models gave order of magnitude agreement with the experimental data. The results of the analysis, Equations (9) and (11), contain coefficients whose value is of the order of unity. Since a value of one gave good agreement with the data, the nature of these coefficients must be examined and estimates made of their most probable value.

The lift coefficient for nonrotating spheres was determined to be unity in a simple analysis in which potential flow was assumed and the sphere was replaced by a simple source and an equal sink (*22*). Data taken by Fage on lift due to the pressure distribution around closely packed pyramids were recalculated by White (*26*) with the result that the lift force was shown to be equal to the drag force. This is equivalent to a ratio of $C_l/C_D = 1/4$. For the present study the drag coefficient based on the Reynolds number calculated with the mean longitudinal velocity in the laminar sublayer was from 4 to 10. This supports the use of a lift coefficient of the order of 1 in Equation (9). However qualitative observations (*26*) with a more loosely packed nonrotating particulate system indicated that the lift coefficient might be much lower than that observed by Fage. It was postulated that pressure equalization due to the looser packing was responsible for the smaller coefficient. Recalculation of Shields' results (*27*) for initiation of bed-load transport gives a value of the lift coefficient of 0.63 for particles having a diameter equivalent to a y^+ less than 10.

For particles in suspension which are free to rotate as they approach the wall an additional lift would be expected owing to the rotation. Studies (*23*) in which a sphere was rotated in a stream having a uniform approach velocity indicated that the lift coefficient of a sphere, rotating with a peripheral velocity five times the approach velocity, might be increased by as much as 60% over the value expected for a nonrotating particle in a stream having a velocity gradient of the magnitude of that found in the laminar sublayer.

The results for the lift coefficient of stationary and rotating particles indicate that the most probable value of the lift coefficient in Equation (9) is between 0.5 and 1.0 with some enhancement expected if the particle is rotating as it approaches the wall. As shown in Figure 8 a value of 1.0 is required to give a good fit to the experimental data.

The phenomenological analysis, in which particle transport was found to occur when the magnitude of the fluctuations at a distance of one particle diameter from the wall equaled the settling rate of the particle, contained the implicit assumption that there was no slip between the particle and the fluid. If there was slip, then a larger value of the fluctuating velocity component would be required. This could be accounted for by using a value for the length term in Equation (11) larger in magnitude than one particle diameter, the value used in calculating curve b of Figure 8.

The ratio of particle to fluid velocity may be estimated from an analysis (28) of the statistical properties of momentum transfer in two-phase flow. For the conditions of the present study the slip ratio was calculated to be 0.97. Hence it is unnecessary to include a slip correction in Equation (11), and the proper magnitude of the length term on the right side of the equation is given by the particle diameter.

CONCLUSIONS

The minimum transport condition for flocculated suspensions may be divided into two categories depending on the concentration. For more concentrated suspensions, in the compaction zone, the flocs are in contact and the suspension acts more or less homogeneously. Under these conditions transport of solids occurs at effective Reynolds numbers corresponding to the end of the transition region where the flow is no longer intermittently laminar and turbulent but is completely turbulent.

For more dilute suspensions, in which the flocs are no longer in contact but are in the hindered-settling zone, the minimum transport condition is the result of a net radial force on the particle away from the wall. For particles smaller than the thickness of the laminar sublayer which settle according to Stokes' law this force results from Bernoulli forces and the turbulent fluctuations which penetrate the laminar sublayer.

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NOTATION

b = exponent, dimensionless
c = concentration, particles per unit volume
C = coefficient, dimensionless
D = tube diameter, ft.
D_p = particle diameter, ft.
f = fanning friction factor, dimensionless
F = force on a particle, lb.-force
g_c = conversion factor, (lb.-mass/lb.-force) (ft./sec.^{2})
g_L = local gravitational acceleration, ft./sec.^{2}
K, n = coefficient, dimensionless
L = tube length, ft.
N_{He} = Hedstrom number, $g_c\ \rho\ \tau_y D^2/\eta^2$, dimensionless
N_{Re} = Reynolds number, $DV\rho/\mu$, $DV\rho/\mu_e$, $DV\rho/\eta$, dimensionless
u = local fluid velocity, ft./sec.
u^+, y^+ = velocity profile co-ordinates, dimensionless
u', v' = turbulent fluctuation components, ft./sec.
u_{*w} = wall shear velocity, ft./sec.
U_t = terminal settling velocity, ft./sec.
V = mean fluid velocity, ft./sec.
y = distance from pipe wall, ft.
Z = vertical distance, ft.
du/dr = velocity gradient, sec.$^{-1}$

Greek Letters

Δp = over-all pressure drop, lb.-force/sq. ft.
ϵ = eddy diffusivity for momentum, sq.ft./sec.
κ = coefficient, dimensionless
η = coefficient of rigidity, lb.-mass/ft. sec.
μ = coefficient of viscosity, lb.-mass/ft. sec.
μ_e = effective viscosity, $\eta[1 + (g_c D\,\tau_y/6\,\eta\,V)]$, lb.-mass/ft. sec.
ρ = fluid density, lb.-mass/cu.ft.
ρ_m = mixture density, lb.-mass/cu. ft.
ρ_p = particle density, lb.-mass/cu. ft.
τ_y = yield stress, lb.-force/sq.ft.
τ_w = wall shear stress, lb.-force/sq.ft.
ϕ = volume fraction solids, dimensionless
Ψ = function of, dimensionless

Subscripts

1 = reference position
2 = reference position
l = lift
g = gravitational
MT = minimum transport condition

LITERATURE CITED

1. Chien, Ning, *Trans. Am. Soc. Civil Engrs.*, **121**, 833-868 (1956).
2. Schlichting, Hermann, "Boundary Layer Theory," Pergamon Press, London (1955).
3. Vanoni, V. A., and N. H. Brooks, *Calif. Inst. of Tech. Sedimentation Lab Report*, E-68 (Dec., 1957).
4. Thomas, D. G., *A.I.Ch.E. Journal*, **6**, No. 4, p. 631 (1960).
5. Durand, R., *Proceedings Minnesota Hydraulic Convention* (1953).
6. Cairns, R. C., *Proc. 2d U. N. Int. Conf. on Peaceful Uses of Atomic Energy*, **7**, *Reactor Technology*, p 45, United Nations Publication (1958).
7. Vine, W. A., ed., *Proceedings of a Symposium on Hydraulic Fill at Montana School of Mines, May 9 and 10, 1958*, Montana School of Mines, Butte, Montana (1958).
8. Overbeeck, J. Th., "Colloid Science," Vol. 1, H. R. Kruyt, ed., Elsevier, Amsterdam (1952).
9. Thomas, D. G., "Fluid Fuel Reactors," p 158-177, J. A. Lane, H. G. MacPherson, and F. Maslan, ed., Addison Wesley, Reading, Mass. (1958).
10. Herdan, G., "Small Particle Statistics," Elsevier, Amsterdam (1953).
11. Einstein, H. A., and Huon Li, *Trans. Am. Soc. Civil Engrs.*, **123**, 293-317 (1958).
12. Einstein, H. A., and E. S. A. El-Samni, *Revs. Modern Phys.*, **21**, 520-524 (1949).
13. Laufer, John, *Natl. Advisory Comm. Aeronaut., 1174* (1954).
14. Boyd, C. M., et al., *Oak Ridge Natl. Lab. Rept., ORNL-2345* (Aug. 5, 1957).
15. Newitt, D. M., et al., *Trans. Inst. Chem. Engrs.*, **33**, 93-110 (1955).
16. Lindgren, E. R., *Arkiv för Fysik*, **16**, 101-112 (1960).
17. Thomas, D. G., *Homogeneous Reactor Project Quar. Prog. Rep. Oct. 31, 1958, ORNL-2654*, p. 69.
18. Steinour, H. H., *Ind. Eng. Chem.*, **36**, 618-624, 840-847, 901-907 (1944).
19. Reed, S. A., and P. R. Crowley, *Nuc. Sci. Eng.*, **1**, 511-521 (1956).
20. Patterson, R. C., *Trans. Am. Soc. Mech. Engrs., J. Eng. Power*, **81**, 43-54 (1959).
21. Murphy, G., D. F. Young, and R. J. Burian, *Ames Laboratory Report*, ISC-474 (April 1, 1954).
22. Munk, Max, *Natl. Advisory Comm. Aeronaut., Rept. 114* (1921).
23. MacColl, J. W., *J. Roy. Aeronaut. Soc.*, **32**, 777 (1928).
24. Fage, A., "50 Years of Boundary Layer Theory," H. Gortler and W. Tollmein, ed., Fred Vieweg & Son, Graunschweig (1955).
25. Deissler, R. G., *Natl. Advisory Comm. Aeronaut., Rept. 1210* (1955).
26. White, C. M., *Proc. Royal Soc. (London)*, **174A**, 322-338 (1940).
27. Shields, A., "Mitteilungen," Preussische Vers. fur Wasserbau und Schiffbau, Berlin (1936).
28. Soo, S. L., *Chem. Eng. Sci.*, **5**, 57-67 (1956).

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